31 Causality and Scattering Amplitudes in Nonlocal Gravity

31 非局部引力中的因果性与散射振幅

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Abstract

摘要

A remarkable feature of weakly nonlocal gravitational theories (eventually coupled to matter), which are compatible with perturbative unitarity and finiteness at quantum level, is that they exhibit the same tree-level scattering amplitudes as Einstein's gravity. After reviewing the explicit computation for the four-graviton scattering amplitude and a general proof for n -point amplitudes based on a field redefinition, we show as a consequence that in nonlocal quantum gravity, a Shapiro's time advance, which is related to a violation of macro-causality, never occurs. Moreover, we provide a recipe to construct a general ultraviolet complete gravitational theory coupled to matter compatible with macro-causality. Finally, we present similar results for local Lee-Wick quantum gravity, whereas we highlight that nonlocal gravity in Weyl basis can potentially violate causality.

弱非局域引力理论 (最终可与物质耦合) 兼容微扰幺正性且在量子层面有限，其一个显著特性是，这类理论展现出与爱因斯坦引力完全相同的树级散射振幅。我们在回顾了四引力子散射振幅的显式计算，以及基于场重定义对 n 点振幅的一般性证明后，推导出结论: 在非局域量子引力中，与宏观因果性破缺相关的夏皮罗时间超前绝不会发生。此外，我们给出了一套构造与宏观因果性兼容、紫外完备的含物质耦合广义引力理论的方案。最后，我们针对局域李-威克量子引力给出了相似结论，同时强调外尔基下的非局域引力有可能违反因果性。

Keywords

关键词

Causality - Scattering amplitudes - Nonlocal gravity

因果性 - 散射振幅 - 非局域引力

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Introduction

引言

The major challenge of quantum gravity lies in the difficulty of reconciling renormalizability and perturbative unitarity. In fact, the Einstein-Hilbert action is not power-counting renormalizable, but, if we introduce the infinite number of counterterms generated by the renormalization procedure, it is perturbatively unitary. On the other hand, it is possible to build higher-derivative theories of quantum gravity that are renormalizable with finitely many counterterms, but are non-unitary. This is the case, for example, of the celebrated Stelle theory [1]. In the literature a number of possible solutions to this puzzle have been discussed. Among them in recent years, nonlocal models received a great deal of attention both for their capability to produce an interesting phenomenology and for their remarkable properties upon quantization. In particular, a class of weakly nonlocal gravitational theories has been proven to be super-renormalizable (or finite) and perturbatively unitary [2-7]. These theories have also been studied in connection to cosmological backgrounds and black hole solutions [8-12].

量子引力的主要挑战在于难以同时兼顾可重整化性与微扰幺正性。事实上，爱因斯坦-希尔伯特作用量并不满足幂次计数可重整化，但如果我们引入重整化过程生成的无穷多抵消项，它是满足微扰幺正性的。另一方面，我们可以构造出仅需有限个抵消项即可实现可重整化的高阶导数量子引力理论，但这类理论不满足幺正性。著名的斯泰勒理论 [1] 就是这类情况的一个例子。文献中已经讨论了该难题的多种可能解决方案。其中，非局域模型近年来受到大量关注，这源于它们能够给出有趣的唯象学，且在量子化后具备出色的性质。特别地，一类弱非局域引力理论已被证明是超可重整化(或有限) 且满足微扰幺正性 [2-7]。这类理论也被结合宇宙学背景与黑洞解开展了研究 [8-12]。

However, the main concern about nonlocal theories is surely causality, whose investigation has only recently been addressed in a systematic way. In a classical theory, the problem is closely related to a mathematically sound formulation of the initial value problem, whereas from the quantum perspective nonlocality implies a new formulation of the Bogoliubov causality condition for local interactions. Remarkably, for quasi-local interactions where the nonlocality shows up only at scales smaller than the nonlocality scale ℓ_Λ , non-causal effects remain confined within the scale ℓ_Λ [13-15]. This has also led to the idea that for asymptotically free nonlocal theories, this violation of microcausality may actually be undetectable [16].

然而，非局域理论最受关注的问题无疑是因果性，该问题直到近年才得到系统研究。在经典理论中，该问题与初值问题的数学严谨表述密切相关；而从量子视角看，非局域性要求我们重新表述局域相互作用的博戈留波夫因果性条件。值得注意的是，对于仅在小于非局域尺度 ℓ_Λ 的范围内显现非局域性的准局域相互作用，非因果效应会被限制在尺度 ℓ_Λ 以内 [13-15]。这也引出了一个观点: 对于渐近自由的非局域理论，这种微观因果性破缺实际上可能无法被探测到 [16]。

In this note, we consider another notion of causality introduced by Gao and Wald [17], according to which it is impossible to send signals faster than what is allowed by the asymptotic causal structure of the spacetime. Its violation has been recently discussed by Camanho, Edelstein, Maldacena, and Zhiboedov [18], in particular in connection to Shapiro's time delay, which is one of the classical tests of general relativity (GR) [19, 20]. Light propagating near a compact object should suffer a time delay compared with the same propagation in flat spacetime. Therefore, if we get a negative time delay, or actually a time advancement, we have a causality violation. Exploiting the relation between the Shapiro time delay and the scattering amplitudes for gravitating particles in the eikonal approximation, the authors of [18] have proven that these causality problems are produced only by the form of the on-shell three-point functions of the theory. Therefore the most general higher derivative gravity theory giving rise to a causality violation is at most cubic in the Riemann tensor. In particular their analysis applies to a high energy scattering process in which gravity is still weakly coupled. This can be achieved if the impact parameter b can be chosen such that

在这篇短文中，我们讨论高与沃尔德提出的另一种因果性概念 [17]，该概念认为信号传播速度不可能超过时空渐近因果结构所允许的上限。卡马尼奥、埃德尔斯坦、马尔达西那和齐博伊多夫最近讨论了该因果性的破缺 [18]，尤其关联到夏皮罗时间延迟——这是广义相对论 (GR) 的经典检验之一 [19,20]。光在致密天体附近传播时，相比于平直时空中的同等传播，会产生时间延迟。因此，如果我们得到负的时间延迟，实际上就是时间超前，就意味着发生了因果性破缺。[18] 的作者利用夏皮罗时间延迟与 eikonal 近似下引力粒子散射振幅的关系，证明了这类因果性问题仅由理论的在壳三点函数的形式导致。因此，会引发因果性破缺的最一般高阶导数引力理论，其里契张量阶数最高为三次。他们的分析尤其适用于引力仍处于弱耦合的高能散射过程。该过程成立的条件是我们可以选取冲击参数 b 满足

$$\ell_{\mathrm{P}} \ll b \ll \ell_{\Lambda}, \tag{1}$$

where ℓ_{P} is the Planck scale and ℓ_{Λ} the nonlocality scale. In such cases the loss of causality can be evaded by adding massive higher spin particles with spin $J > 2$ and mass $m^2 \sim \ell_{\Lambda}^{-2}$.

其中 ℓ_{P} 是普朗克尺度，ℓ_{Λ} 是非局域尺度。在这类情况中，可以通过加入自旋为 $J > 2$ 、质量为 $m^2 \sim \ell_{\Lambda}^{-2}$ 的大质量高自旋粒子来避免因果性丧失。

In the following we want to argue that the terms responsible for this causality violation do not need to show up in a nonlocal theory of quantum gravity of the kind studied in [2-7]. So these theories turn out to be consistent even without the introduction of an infinite tower of massive higher spin fields. First, in section ”Weakly Nonlocal Gravity,” we review some essential features of the class of theories under consideration, in particular their renormalization properties and perturbative unitarity. Then, in section ”Scattering Amplitudes in Higher Derivative Gravity Theories” we review results about scattering amplitudes in weakly nonlocal theories pointing out the crucial role played by a theorem relating tree-level amplitudes in theories related by field redefinitions [25,26]. In section ”Shapiro’s Time Delay,” we finally report about the results of [27], where the problem of causality has been addressed.

在下文中我们将说明，引发这种因果性破缺的项并不一定会出现在 [2-7] 所研究的这类非局域量子引力理论中。因此，即使不引入无穷多的大质量高自旋场，这类理论也是自洽的。首先，我们在“弱非局域引力”一节回顾了所讨论的这类理论的核心性质，尤其是它们的重整化性质与微扰幺正性。随后，我们在“高阶导数引力理论中的散射振幅”一节回顾了弱非局域理论中散射振幅的相关结果，指出了联系场重定义等价理论树图振幅的定理起到了关键作用 [25,26]。最后在“夏皮罗时间延迟”一节，我们介绍了 [27] 中针对因果性问题的研究结果。

Weakly Nonlocal Gravity

弱非局域引力

We investigate the class of theories defined by the action

我们研究由如下作用量定义的这类理论

$$S_g = \frac{2}{\kappa_D^2} \int d^D x \sqrt{-g} \left[R + G_{\mu\nu} \gamma(\Box) R^{\mu\nu} + V(\mathcal{R}) \right], \tag{2}$$

where $\kappa_D^2 = 32\pi G$. Given the nonlocality scale $\sigma \equiv \ell_\Lambda^2$, the form factor $\gamma(\Box)$ is defined by

其中 $\kappa_D^2 = 32\pi G$ 。给定非局域尺度 $\sigma \equiv \ell_\Lambda^2$ ，形状因子 $\gamma(\Box)$ 定义为

$$\gamma(\Box) = \frac{e^{H(\sigma\Box)} - 1}{\Box}, \tag{3}$$

where the function $\exp H(z)$ is asymptotically polynomial in a conical region C around the real axis, namely,

其中函数 $\exp H(z)$ 在实轴周围的锥形区域 C 内渐近为多项式，即:

$$|\exp H(z)| \to |z|^{\gamma+\mathrm{N}+1} \text{ for } |z| \to +\infty, \tag{4}$$

with N an integer defined in terms of the spacetime dimension D so that $2N + 4 = D$ (if D is even) or $2N + 4 = D + 1$ (if D is odd). This condition is necessary to avoid the appearance of nonlocal counterterms in the UV regime. An example due to Tomboulis [3] is

其中 N 是由时空维度 D 定义的整数，因此当 D 为偶数时满足 $2N + 4 = D$ ，当 D 为奇数时满足 $2N + 4 = D + 1$ 。该条件是避免紫外区出现非局域抵消项的必要条件。Tomboulis[3] 给出的一个例子是

$$H_T(z) = \frac{1}{2} \left[\Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right) \right], \tag{5}$$

where $p(z)$ is a polynomial of degree $\gamma + \mathrm{N} + 1, \Gamma(a, z)$ the incomplete Gamma function, and γ_E the Euler-Mascheroni constant. The local potential $V(\mathcal{R})$ is at least cubic in the curvature, namely, $V \sim O\left(\mathcal{R}^3\right)$, but quadratic in the Ricci tensor, and is taken to contain at most $2\gamma + 2N + 4$ derivatives. These choices are motivated by inspection of quantum divergence in the UV regime. In fact the graviton propagator scales as $k^{-(2\gamma+2N+4)}$ and the vertices contain terms whose leading behavior is just the inverse. This determines the upper bound on the superficial degree of divergence of any graph $G, \omega(G) \equiv DL + (V - I)(2\gamma + 2N + 4)$. We find in a spacetime of even or odd dimension, respectively,

其中 $p(z)$ 是次数为 $\gamma + \mathrm{N} + 1, \Gamma(a, z)$ 的多项式，γ_E 是欧拉-马歇罗尼常数。局域势 $V(\mathcal{R})$ 对曲率而言至少是三次的，即 $V \sim O\left(\mathcal{R}^3\right)$ ，但对里奇张量而言是二次的，且最多包含 $2\gamma + 2N + 4$ 阶导数。这些选择是通过分析紫外区的量子发散得到的。实际上引力子传播子的标度为 $k^{-(2\gamma+2N+4)}$ ，顶点中各项的领头行为恰好是其逆。这就确定了任意图 $G, \omega(G) \equiv DL + (V - I)(2\gamma + 2N + 4)$ 表面发散度的上界。我们分别在偶数维与奇数维时空下得到:

$$\omega(G)_{\text{even}} = D_{\text{even}} - 2\gamma(L - 1), \ \omega(G)_{\text{odd}} = D_{\text{odd}} - (2\gamma + 1)(L - 1). \tag{6}$$

Thus, if $\gamma > D_{\text{even}}/2$ or $\gamma > (D_{\text{odd}} - 1)/2$, only one-loop divergences survive. Therefore, the theory is super-renormalizable, and only a finite number of operators of mass dimension up to M^D has to be included in the action for renormalization in even dimensions. In odd dimensions, due to dimensional reasons, there are no divergences at one loop, and the theory is automatically finite. The freedom in the choice of the potential $V(\mathcal{R})$ can be used to "kill" the one-loop divergences in even dimensions. For example, in four dimensions, it is possible to prove that the two quartic killer operators

> 因此，若满足 $\gamma > D_{\text{even}}/2$ 或 $\gamma > (D_{\text{odd}} - 1)/2$，则仅单圈发散保留。因此该理论是超可重整的，在偶数维中仅需在作用量中加入有限个质量维度不超过 M^D 的算符即可完成重整化。在奇数维中，因维度原因，单圈不存在发散，理论自动有限。我们可以利用势 $V(\mathcal{R})$ 的选择自由度“消除”偶数维中的单圈发散。例如，可以证明在四维中，两个四次“杀伤算符”

$$
s_1 R^2 \Box^{\gamma-2} R^2 + s_2 R_{\mu\nu} R^{\mu\nu} \Box^{\gamma-2} R_{\rho\sigma} R^{\rho\sigma} \tag{7}
$$

give contributions to the beta functions of the couplings for R^2 and $R_{\mu\nu}^2$ which are linear in their front coefficients s_1 and s_2 so that finiteness can be achieved by choosing them so that $\beta_{R^2} = \beta_{R_{\mu\nu}^2} = 0$. The crucial point for the following is that the killer terms should be in general at least quadratic in the Ricci tensor. One can easily find from the kinetic term the two-point function in the harmonic gauge $\left(\partial^\mu h_{\mu\nu} = 0\right)$,

> 会对 R^2 和 $R_{\mu\nu}^2$ 的耦合 β 函数产生贡献，贡献与它们的前系数 s_1 和 s_2 呈线性关系，因此只要选择系数满足 $\beta_{R^2} = \beta_{R_{\mu\nu}^2} = 0$ 就能得到有限理论。下文的核心要点是: 杀伤项通常至少对里奇张量是二次的。我们可以很容易从动能项得到调和规范 $\left(\partial^\mu h_{\mu\nu} = 0\right)$, 下的两点传播子

$$
\mathcal{O}^{-1} \sim \frac{1}{k^2 e^{H(k^2/\Lambda^2)}} \left(P^{(2)} - \frac{P^{(0)}}{D-2} \right), \tag{8}
$$

where $P^{(0)}$ and $P^{(2)}$ are the usual spin 2 and spin 0 projectors. Therefore perturbative unitarity together with the absence of gauge invariant poles other than the graviton pole requires that $\exp H(z)$ be real and positive on the real axis and without zeros on the whole complex plane $|z| < +\infty$. This choice however implies a subtlety related to the fact that amplitutes are well-defined as integrals along certain loop integration contours and in a certain regime of external momenta, which is typically the Euclidean one. The vertices we have defined in order to achieve UV finiteness must decrease sufficiently fast along some directions in the complex plane, namely, the ones corresponding to Euclidean momenta. However, for the nonpolynomial entire functions, this necessarily implies a fast growth in other directions in the complex plane, thus generally preventing the usual Wick rotation. This could generically point at a violation of perturbative unitarity. However, the theories considered in this note turn out unitary at perturbative level to all perturbative orders in the loop expansion as rigorously and extensively proved in [21] and more recently in [22-24]. The proof is based on an analytic continuation of the external particles' energies from imaginary to real values. It turns out that the Landau singularities and the discontinuities of the amplitudes are the same of a local theory at any perturbative order in the loop expansion. This is a consequence of the classical spectrum of the theory that is the same of the local theory. Therefore, the Cutkosky cutting rules are the same of the local theory. Finally, there is no contribution of cut diagrams corresponding to anomalous thresholds to the imaginary part of the scattering amplitudes as proved in [22, 24]. Indeed, if a diagram is cut in less than two or more than two parts, the contribution to the discontinuities vanishes as a consequence of the energy momentum conservation.

其中 $P^{(0)}$ 和 $P^{(2)}$ 是常规的自旋 2 投影算符和自旋 0 投影算符。因此，微扰幺正性要求，除引力子极点外不存在其他规范不变极点，即 $\exp H(z)$ 在实轴上为正实数，且在整个复平面 $|z| < +\infty$ 上无零点。然而该选择存在一个微妙之处: 振幅只有沿特定圈积分围道、在特定外动量区域 (通常是欧几里得区域) 才能得到良定义。我们为实现紫外有限性定义的顶点，必须沿复平面的部分方向 (即对应欧几里得动量的方向) 足够快地衰减。但对于非多项式整函数而言，这必然意味着其在复平面的其他方向快速增长，因此通常无法进行常规威克转动。这一般会指向微扰幺正性破坏。然而，本文研究的理论在圈展开的所有微扰阶下都是微扰幺正的，这一点已在文献 [21] 中得到严格充分的证明，近期文献 [22-24] 也给出了证明。该证明基于将外粒子能量从虚值解析延拓到实值。可以证明，朗道奇点与振幅间断在圈展开的任意微扰阶都和定域理论完全一致，这是该理论的经典能谱与定域理论一致的结果。因此，卡特斯基切割规则也和定域理论一致。最后，如文献 [22, 24] 所证，散射振幅虚部不存在反常阈值对应的切割图贡献。事实上，如果一个图被切割为少于两部分或多于两部分，由于能量动量守恒，其对间断的贡献为零。

Scattering Amplitudes in Higher Derivative Gravity Theories

高导数引力理论中的散射振幅

We here review some results about scattering amplitudes in higher derivative gravity theories, in particular four-graviton tree-level ones in the case when all higher derivative terms are at least quadratic in the Ricci tensor. For the sake of clarity, we first consider an action

我们在此回顾高导数引力理论中散射振幅的一些研究结果，尤其是当所有高导数项在里奇张量中至少为二次项时，四点引力子的树级散射振幅结果。为清晰起见，我们首先考虑如下作用量

$$S_g = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\big(R + R\gamma_0 R + R_{\mu\nu}\gamma_2 R^{\mu\nu}$$

$$+\big(R_{\mu\nu\rho\sigma}\gamma_4 R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\gamma_4 R^{\mu\nu} + R\gamma_4 R\big)\big), \tag{9}$$

where γ_0, γ_2 , and γ_4 are generic functions of $\sigma\Box$. As observed in [25], in such cases the computation can be addressed by standard Feynman diagram techniques due to a number of simplifications. First of all, we note the last term is the famous Gauss-Bonnet density, which is topological in four dimensions, whereas for generic higher dimensions, it gives rise to vertices only. Furthermore, as the process involves only three-graviton vertices with two gravitons on-shell and one off-shell and a four-graviton vertex with all external legs on-shell, we can make full use of the linearized vacuum equation of motion for the physical field $h_{\mu\nu}$ in the harmonic gauge, i.e., $\Box h_{\mu\upsilon} = 0$. Actually, we can choose polarizations satisfying the conditions $\partial^\mu h_{\mu\upsilon} = h^\mu_\mu = 0$ all along the computation, which greatly simplifies the algebra. In fact, these conditions imply that all the scalar operators are vanishing on-shell at linear order in $h_{\mu\nu}$, including the scalar curvature $R^{(1)}$ and the root of metric determinant $\sqrt{-g}^{(1)}$. One can further show that $R^{(1)}_{\mu\nu} = 0$ due to the linearized EOM. We can express all the amplitudes in terms of the Mandelstam variables $s = 4E^2, t = -2E^2(1-\cos\theta)$ and $u = -2E^2(1+\cos\theta)$, with E the energy and θ the scattering angle in the center-of-mass reference frame.

其中 γ_0, γ_2 和 γ_4 是 $\sigma\square$ 的任意函数。正如文献 [25] 指出的，在这类情形中，由于存在多个简化条件，我们可以通过标准费曼图技术完成计算。首先，我们注意到最后一项是著名的高斯-博内密度，它在四维空间中是拓扑项；而在任意更高维度中，它仅会贡献顶角。此外，由于该过程仅涉及两个引力子在壳、一个引力子离壳的三点引力子顶角，以及所有外腿都在壳的四点引力子顶角，我们可以充分利用谐规范下物理场 $h_{\mu\nu}$ 的线性化真空运动方程，即 $\square h_{\mu\nu} = 0$。实际上，我们可以在整个计算过程中选择满足条件 $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$ 的极化，这能大幅简化代数运算。事实上，这些条件意味着所有标量算符在 $h_{\mu\nu}$ 的线性阶上都在壳消失，包括标量曲率 $R^{(1)}$ 和度规行列式的根 $\sqrt{-g}^{(1)}$。还可以进一步证明，根据线性化运动方程可得 $R^{(1)}_{\mu\nu} = 0$。我们可以用曼德尔斯坦变量 $s = 4E^2, t = -2E^2(1-\cos\theta)$ 和 $u = -2E^2(1+\cos\theta)$ 表示所有散射振幅，其中 E 是质心系中的能量，θ 是质心系中的散射角。

In the case where γ_0, γ_2 , and γ_4 are constants, for gravitons with positive helicity, one finds for $D = 4, 5, 6$

当 γ_0, γ_2 和 γ_4 为常数时，对于正螺旋度的引力子，可得 $D = 4, 5, 6$ 满足

$$\mathcal{A}^{D=4}(++,++) = -i\frac{1}{\kappa_4^2}\frac{E^2}{\sin^2\theta}, \tag{10}$$

$$\mathcal{A}^{D=5}(++,++) = i\frac{1}{\kappa_5^2}\left\{\frac{8E^6\gamma_4^2\left[1+8E^2(3\gamma_0+\gamma_2)\right]}{(1-4E^2\gamma_2)\left[3+4E^2(16\gamma_0+5\gamma_2)\right]} - \frac{E^2}{\sin^2\theta}\right\}, \tag{11}$$

$$\mathcal{A}^{D=6}(++,++) = i\frac{1}{\kappa_6^2}\left\{\frac{4E^6\gamma_4^2\left[1+8E^2(3\gamma_0+\gamma_2)\right]}{(1-4E^2\gamma_2)\left[1+2E^2(10\gamma_0+3\gamma_2)\right]} - \frac{E^2}{\sin^2\theta}\right\}. \tag{12}$$

Remarkably, in four dimensions, where γ_4 cannot enter the amplitude because of the Gauss-Bonnet theorem, the result coincides with the one expected in Einstein theory by dimensional analysis and symmetry arguments. In particular no term scaling as E^4 in the UV shows up as it would be natural to expect in a four-derivative theory. This is the result of non-trivial cancellations between the massive poles in the propagator and the three-graviton vertices and between the contact and exchange diagrams. This result can be also understood as the one consistent with the natural expectation in the limit where the Einstein term can be dropped out only leaving the scaleless quadratic terms. They would be expected to naively give amplitudes $\sim E^4$, but this cannot happen because the graviton field is dimensionless and there is no other scale. So the amplitude is actually expected to vanish. In $D > 4, \gamma_4$ enters the amplitude, but only quadratically, whereas the expected linear dependence is absent due to a cancellation between the contact diagram with vertex from Gauss-Bonnet term and the exchange diagrams with two different vertices (one from GB, the second one from standard terms R, R^2 , or $R^2_{\mu\nu}$). Whereas in the ultraviolet regime the amplitude scales as E^4 , in the infrared one finds arbitrary powers of E^2 associated with the massive poles in the propagators which cannot cancel with the three-graviton vertices of the Gauss-Bonnet density.

值得注意的是，根据高斯-博内定理，四维情况下 γ_4 不会出现在振幅中，所得结果与通过量纲分析和对称性论证得到的爱因斯坦理论预期结果完全一致。特别地，紫外区域并没有出现自然预期的、正比于 E^4 的项——这源于传播子中质量极点与三点引力子顶角之间，以及接触图和交换图之间的非平凡抵消。该结果也符合如下自然预期: 当爱因斯坦项可以忽略，仅留下无标度二次项时，naive 来看人们会预期这些项给出振幅 $\sim E^4$ ，但实际不可能，因为引力子场是无量纲的，且不存在其他尺度，因此振幅实际应当为零。在 $D > 4, \gamma_4$ 维中，γ_4 会进入振幅，但仅以二次形式出现，预期的线性依赖由于抵消消失: 来自高斯-博内项顶角的接触图，与带有两个不同顶角 (一个来自高斯-博内项，一个来自标准项 R, R^2 或 $R_{\mu\nu}^2$) 的交换图发生了抵消。紫外区域振幅正比于 E^4 scaling，而红外区域则存在任意幂次的 E^2 ，这些项对应传播子中的质量极点，无法与高斯-博内密度的三点引力子顶角发生抵消。

For weakly nonlocal gravity, the amplitude can be also performed straightforwardly if $\gamma_4(\Box) = 0$. In fact, as on-shell $\mathbf{R} \sim O(h^2)$ and Ric $\sim O(h^2)$ (whereas Riem $\sim O(h)$), the form factors are spectators in the expansion in the number of gravitons, and many results for the Stelle gravity apply to the general nonlocal theory. For the three exchange diagrams and the contact one, we find

对于弱非局域引力，若 $\gamma_4(\Box) = 0$ ，我们也可以直接计算振幅。事实上，由于在壳的 $\mathbf{R} \sim O(h^2)$ 和 Ric $\sim O(h^2)$ (而 Riem $\sim O(h)$)，形状因子在引力子数量展开中不参与变化，因此 Stelle 引力的许多结果都适用于一般非局域理论。对于三个交换图和一个接触图，我们得到

$$\mathcal{A}_s(++,++) = \frac{i}{16\kappa_4^2}\left(-9\frac{t(s+t)}{s} + \frac{9}{4}\gamma_2(s)\left(s^2 + (s+2t)^2\right) + 9s^2\gamma_0(s)\right), \tag{13}$$

$$\begin{aligned}\mathcal{A}_t(++,++) = \frac{i}{16\kappa_4^2}\Bigg(&-\frac{\left(s^3 - 5s^2t - st^2 + t^3\right)(s+t)^2}{s^3 t} \\ &+\frac{1}{2}\gamma_2(t)\frac{\left(2s^4 - 10s^3t - s^2t^2 + 4st^3 + t^4\right)(s+t)^2}{s^4} \\ &+\gamma_0(t)\frac{t^2(s+t)^4}{s^4}\Bigg),\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{A}_u(++,++) = \frac{i}{16\kappa_4^2}\Bigg(&-\frac{\left(s^3 - 5s^2u - su^2 + u^3\right)(s+u)^2}{s^3 u} \\ &+\frac{1}{2}\gamma_2(u)\frac{\left(2s^4 - 10s^3u - s^2u^2 + 4su^3 + u^4\right)(s+u)^2}{s^4} \\ &+\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\Bigg),\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{A}_{\text{contact}}(++,++) = \frac{i}{16\kappa_4^2}\Bigg(&-2\frac{s^4 + s^3t - 2st^3 - t^4}{s^3} - \frac{9}{4}\gamma_2(s) \\ &\times\left(s^2 + (s+2t)^2\right) - 9s^2\gamma_0(s) \\ &-\frac{1}{2}\gamma_2(t)\frac{\left(2s^4 - 10s^3t - s^2t^2 + 4st^3 + t^4\right)(s+t)^2}{s^4}\end{aligned}$$

$$-\gamma_0(t)\frac{t^2(s+t)^4}{s^4}$$

$$-\frac{1}{2}\gamma_2(u)\frac{\left(2s^4-10s^3u-s^2u^2+4su^3+u^4\right)(s+u)^2}{s^4}$$

$$-\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\Bigg), \tag{16}$$

where the possible poles associated with γ_0 and γ_2 cancel separately in each channel. Once again, the amplitude

其中与 γ_0 和 γ_2 相关的可能极点在每个通道中分别抵消。再一次，振幅

$$\mathcal{A}(++,++) = \mathcal{A}_s(++,++) + \mathcal{A}_t(++,++) + \mathcal{A}_u(++,++)$$

$$+\mathcal{A}_{\text{contact}}(++,++) = \mathcal{A}(++,++)_{\text{EH}}, \tag{17}$$

coincides with the one in Einstein-Hilbert theory.

与爱因斯坦-希尔伯特理论中的结果一致。

These results, which may look somewhat surprising, find actually a very natural explanation in terms of a field redefinition theorem [25,26] that allows to map nonlocal field theories to local ones at tree-level.

这些结果看似出人意料，实际上可以通过场重定义定理 [25,26] 得到非常自然的解释: 该定理允许在树图级别将非局域场论映射为局域场论。

In particular, let us consider two general weakly nonlocal actions $S'(g,\Phi_a)$ and $S(g',\Phi'_a)$, respectively, defined in terms of the fields g,Φ_a and g',Φ'_a , where g is the metric and Φ_a a set of matter or gauge fields and such that

具体而言，我们考虑两个一般的弱非局域作用量 $S'(g,\Phi_a)$ 和 $S(g',\Phi'_a)$, 它们分别由场 g,Φ_a 和 g',Φ'_a 定义，其中 g 是度规，Φ_a 是一组物质场或规范场，且满足

$$S'(g,\Phi_a) = S(g,\Phi_a) + E_i^g(g,\Phi_a)F_{ij}^g(g,\Phi_a)E_j^g(g,\Phi_a)$$

$$+E_a^\Phi(g,\Phi_c)F_{ab}^\Phi(g,\Phi_c)E_b^\Phi(g,\Phi_c), \tag{18}$$

where F^g and F^Φ can contain derivative operators or weakly nonlocal operators of the covariant $\Box$ operator, and

其中 F^g 和 F^Φ 可以包含协变 $\Box$ 算符的导数算符或弱非局域算符，且

$$E_i^g = \frac{\delta S}{\delta g_i},\; E_a^\Phi = \frac{\delta S}{\delta \Phi_a} \tag{19}$$

are the EOM of the theory with action $S(g,\Phi_a)$. The statement of the theorem is that there exists a field redefinition

是作用量为 $S(g,\Phi_a)$ 的理论的运动方程。该定理指出，存在一个场重定义

$$g_i' = g_i + \Delta_{ij}^{g} E_j^{g},\ \Delta_{ij}^{g} = \Delta_{ji}^{g},$$

$$\Phi_a' = \Phi_a + \Delta_{ab}^{\Phi} E_b^{\Phi},\ \Delta_{ab}^{\Phi} = \Delta_{ba}^{\Phi}, \tag{20}$$

such that, perturbatively in $F^{g,\Phi}$, but to all orders in powers of $F^{g,\Phi}$, we have the equivalence

使得在 $F^{g,\Phi}$ 的微扰下，直到 $F^{g,\Phi}$ 幂次的所有阶，我们都能得到等价关系

$$S'(g,\Phi) = S(g',\Phi'). \tag{21}$$

The indices i, a encode all Lorentz and group indices, as well as the spacetime dependence of the fields. Δ_{ij}^{g} (Δ_{ab}^{Φ}) could be a weakly nonlocal or quasi-polynomial operator acting linearly on the EOM E_j^{g} (E_a^{Φ}), and they are defined perturbatively in powers of the operators $F^{g,\Phi}$, namely,

指标 i, a 编码了所有洛伦兹指标、群指标以及场的时空依赖。Δ_{ij}^{g} (Δ_{ab}^{Φ}) 可以是作用在运动方程 E_j^{g} (E_a^{Φ}) 上的线性弱非局域算符或拟多项式算符，它们是按算符 $F^{g,\Phi}$ 的幂次微扰定义的，即

$$\Delta_{ij}^{g} = F_{ij}^{g} + \ldots \quad \text{or} \quad \Delta_{ab}^{\Phi} = F_{ab}^{\Phi} + \ldots. \tag{22}$$

The claim above can be straightforwardly checked at the first order in the Taylor expansion for the functional $S(g',\Phi_a')$

上述结论可以直接在泛函 $S(g',\Phi_a')$ 泰勒展开的一阶验证

$$S(g',\Phi_a') = S(g+\delta g,\Phi+\delta\Phi) \approx S(g) + \frac{\delta S}{\delta g_i}\delta g_i + \frac{\delta S}{\delta \Phi_a}\delta\Phi_a$$

$$= S(g) + E_i^{g}\delta g_i + E_a^{\Phi}\delta\Phi_a, \tag{23}$$

which is consistent with the equivalence (21) if we assume the field redefinitions (20) with the chosen coefficients (22). The theorem states the equivalence of the two theories only perturbatively in $F^{g,\Phi}$, so that the two theories do not need to be equivalent in all aspects. For example, $S'(g,\Phi)$ can have additional poles in the spectrum compared with $S(g',\Phi')$, and also the quantum behaviors of the theories can be completely different. However, the theorem applies to all the n-points tree-level functions whose external legs are on the mass-shell shared by the two theories, and this explains the results found by direct computation for theories that are quadratic in both the Ricci and scalar curvature and lack a term quadratic in the Riemann tensor.

若我们假设场重定义 (20) 采用选定的系数 (22)，则结果与等价关系 (21) 自洽。该定理仅表明两个理论在 $F^{g,\Phi}$ 的微扰下等价，因此两个理论无需在所有方面都等价。例如，$S'(g,\Phi)$ 的谱中可以存在比 $S(g',\Phi')$ 更多的额外极点，两个理论的量子行为也可以完全不同。但该定理适用于所有外腿在两个理论共同质量壳上的 n 点树图函数，这就解释了对里奇曲率、标量曲率都是二次型且不含黎曼张量二次项的理论，直接计算得到的上述结果。

Shapiro's Time Delay

夏皮罗时间延迟

The results of the previous section can be nicely translated in the language of Shapiro's time delay. This can in fact be recovered from the scattering amplitudes in the so-called eikonal approximation, which resumes a particular set of diagrams (horizontal ladders) in the deflectionless limit $t/s << 1.s$ is large compared to the inverse of the nonlocality scale ℓ_Λ^{-2} , but still well below the Planck scale so that the theories we are considering are still weakly coupled. Under favorable circumstances, the amplitude exponentiates in the impact parameter space [28, 29]

上一节的结果可以很好地转换到夏皮罗时间延迟的框架中。实际上这可以从所谓程函近似下的散射振幅得到，该近似在无偏折极限下重整了一组特殊的图 (水平梯形图): $t/s << 1.s$ 远大于非定域性标度的倒数 ℓ_Λ^{-2} ，但仍远低于普朗克尺度，因此我们研究的理论仍处于弱耦合。在合适的条件下，振幅在碰撞参数空间 [28, 29] 中指数化

$$\mathcal{A}_{\text{eik}} = 2s \int d^{D-2}\vec{b} e^{-i\vec{q}\cdot\vec{b}} \left[e^{i\delta(b,s)} - 1\right], \tag{24}$$

where the phase is given by

其中相位由下式给出

$$\delta(b,s) = -\frac{i}{2s} \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}} \mathcal{A}_{\text{tree}}\left(s, -\vec{q}^2\right). \tag{25}$$

Shapiro's time delay is then given by

随后夏皮罗时间延迟可表示为

$$\Delta t = 2\partial_E \delta(E,b). \tag{26}$$

where E is the energy of the probe particle.

其中 E 是探测粒子的能量。

In particular, for the action (9), with $\gamma_0 = \gamma_2 = 0$ and $\gamma_4 = \lambda_{GB}$ a constant, one finds for the four-graviton amplitude in the Regge limit [30]

特别地，对于作用量 (9)，当 $\gamma_0 = \gamma_2 = 0$ 和 $\gamma_4 = \lambda_{GB}$ 为常数时，可得雷杰极限下四引力子振幅为 [30]

$$A_t = \mathcal{A}_{t\mathrm{EH}} + \mathcal{A}_{t\mathrm{GB}}$$

$$\approx -i\frac{8\pi G s^2}{t}\left(\varepsilon_1 \cdot \varepsilon_3\right)\left(\varepsilon_2 \cdot \varepsilon_4\right) + i\frac{8\pi G \lambda_{\mathrm{GB}} s^2}{t}$$

$$\times \left(k_2^\mu k_4^\upsilon \varepsilon_{2\upsilon}^\rho \varepsilon_{4\rho\mu} \varepsilon_1 \cdot \varepsilon_3 + k_1^\mu k_3^\upsilon \varepsilon_{1\upsilon}^\rho \varepsilon_{3\rho\mu} \varepsilon_2 \cdot \varepsilon_4\right),$$

where $\kappa_D^2 = 8\pi G$; the momenta of the four gravitons k_1, k_2, k_3 , and k_4 are all incoming, i.e., $\sum_{i=1}^{4} k_i = 0$; and $\varepsilon_i\,(i = 1, \ldots, 4)$ are the polarizations of the gravitons. Choosing the metric $ds^2 = -dudv + \sum_{i=1}^{D-2} \left(dx^i\right)^2$, we can evaluate this amplitude in the following momentum configuration:

其中 $\kappa_D^2 = 8\pi G$；四个引力子的动量 k_1, k_2, k_3 和 k_4 全部为入射，即满足 $\sum_{i=1}^{4} k_i = 0$；$\varepsilon_i\,(i = 1, \ldots, 4)$ 为引力子的偏振。选定度规 $ds^2 = -dudv + \sum_{i=1}^{D-2} \left(dx^i\right)^2$ 后，我们可以在以下动量构型下计算该振幅:

$$k_{1\mu} = \left(k_u, \frac{\vec{q}^2}{16k_u}, \frac{\vec{q}}{2}\right),\ k_{3\mu} = -\left(k_u, \frac{\vec{q}^2}{16k_u}, -\frac{\vec{q}}{2}\right)$$

$$k_{2\mu} = \left(\frac{\vec{q}^2}{16k_\upsilon}, k_\upsilon, -\frac{\vec{q}}{2}\right),\ k_{4\mu} = -\left(\frac{\vec{q}^2}{16k_\upsilon}, k_\upsilon, \frac{\vec{q}}{2}\right) \tag{27}$$

$$s \simeq 4k_u k_\upsilon,\ t \simeq -\left(\vec{q}\right)^2,$$

where we just kept the leading order in the t/s expansion, assuming $t/s \gg 1$. We also take the polarizations $\varepsilon^{\mu\nu} = \varepsilon^\mu \varepsilon^\upsilon$, given by

我们仅保留了 t/s 展开的领头阶，假设条件为 $t/s \gg 1$。同时我们取偏振 $\varepsilon^{\mu\nu} = \varepsilon^\mu \varepsilon^\upsilon$，由下式给出

$$\varepsilon_1^\mu = \left(-\frac{\vec{q} \cdot \vec{e}_1}{2k_u}, 0, \vec{e}_1\right),\ \varepsilon_3^\mu = \left(\frac{\vec{q} \cdot \vec{e}_3}{2k_u}, 0, \vec{e}_3\right) \tag{28}$$

$$\varepsilon_2^\mu = \left(0, \frac{\vec{q} \cdot \vec{e}_2}{2k_\upsilon}, \vec{e}_2\right),\ \varepsilon_4^\mu = \left(0, -\frac{\vec{q} \cdot \vec{e}_4}{2k_\upsilon}, \vec{e}_4\right). \tag{29}$$

Choosing $e_1 = e_3$ and $e_2 = e_4$, we can compute the phase (25) for the Einstein-Hilbert term

选定 $e_1 = e_3$ 和 $e_2 = e_4$ 后，我们可以计算爱因斯坦-希尔伯特项对应的相位 (25)

$$\delta_{\mathrm{g}}(b, s) = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{Gs}{b^{D-4}} \left(e_1 \cdot e_3\right)\left(e_2 \cdot e_4\right), \tag{30}$$

and for the Gauss-Bonnet term

以及高斯-博内项对应的相位

$$\delta_{\mathrm{GB}}(b,s) = 4\lambda_{\mathrm{GB}}\left(\left(e_1^{ij}e_1^{ij}\right)e_2^{ij}e_2^{ik} + \left(e_2^{ij}e_2^{ij}\right)e_1^{ij}e_1^{ik}\right)\partial_{b_i}\partial_{b_j}\frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{Gs}{b^{D-4}}$$

$$= -4\lambda_{\mathrm{GB}}\frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{Gs}{b^{D-2}}$$

$$\times\left[2(e_1\cdot e_1)(e_2\cdot e_2) - (D-2)(n\cdot e_1)^2 - (D-2)(n\cdot e_2)^2\right], \quad (31)$$

where $\vec{n} \equiv \vec{b}/b$. The total contribution to the phase is given by the sum of (30) and (31), namely,

其中 $\vec{n} \equiv \vec{b}/b$ 。相位的总贡献为 (30) 和 (31) 之和，即:

$$\delta_{\mathrm{g-GB}}(b,s) = \delta_{\mathrm{g}}(b,s) + \delta_{\mathrm{GB}}(b,s)\,. \quad (32)$$

Finally, the Shapiro's time delay is

最终，夏皮罗时间延迟为

$$\Delta t_{\mathrm{g-GB}} = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{16EG}{b^{D-4}}(e_1\cdot e_1)(e_2\cdot e_2)$$

$$\times\left[1 + \frac{4\lambda_{\mathrm{GB}}(D-2)(D-4)}{b^2}\left(\frac{(n\cdot e_1)^2}{e_1\cdot e_1} + \frac{(n\cdot e_2)^2}{e_2\cdot e_2} - \frac{2}{D-2}\right)\right]. \quad (33)$$

We can see that if the impact factor b^2 becomes small, $b^2 < \lambda_{\mathrm{GB}}$, the third term in (33) can be bigger than the first two, depending on the sign of λ_{GB} and the polarizations. Therefore, we can have a time advance, and causality is violated.

我们可以看到，如果碰撞因子 b^2 变小，即 $b^2 < \lambda_{\mathrm{GB}}$ ，(33) 式的第三项会大于前两项，具体大小取决于 λ_{GB} 的符号和偏振。因此，我们可能得到时间超前，因果性被破坏。

On the other hand, we saw in the previous sections that for theories that are quadratic in both the Ricci and scalar curvature and lack a term quadratic in the

另一方面，我们在前几节已经看到，对于里奇曲率和标量曲率均为二次、且不含

Riemann tensor, the tree-level amplitude exactly coincides with the Einstein-Hilbert one. The corresponding time delay is

黎曼张量二次项的理论，树 level 振幅与爱因斯坦-希尔伯特理论的结果完全一致，对应的时间延迟为

$$\Delta t_{\mathrm{g}} = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{16EG}{b^{D-4}}(e_1\cdot e_3)(e_2\cdot e_4)\,, \quad (34)$$

and of course no time advancement is possible.

当然也就不可能出现时间超前。

Actually, any nonlocal theory that is tree-level equivalent by the field redefinition theorem to a causal local one is causal too. In other words, given a causal (possibly local) theory, the theorem provides an algorithm for constructing a full class of higher derivative (even nonlocal) causal theories.

实际上，任何通过场重定义定理与因果局域理论在树图层级等价的非局域理论也同样是因果的。换言之，给定一个因果 (可能是局域的) 理论，该定理提供了一套构造完整高阶导数 (甚至非局域) 因果理论类的算法。

An explicit example of a nonlocal theory involving gravity, one gauge field, and a scalar field is the one given by the action

一个包含引力、一个规范场和一个标量场的非局域理论显例由如下作用量给出

$$
\begin{aligned}
\mathcal{L} = \frac{1}{2\kappa_D^2}\left[R + \left(G_{\mu\upsilon} - 2\kappa_D^2\left(T_{\mu\upsilon}^A + T_{\rho\sigma}^{\phi}\right)\right)F_{\mathrm{g}}^{\mu\upsilon,\rho\sigma}\left(G_{\rho\sigma} - 2\kappa_D^2\left(T_{\rho\sigma}^A + T_{\rho\sigma}^{\phi}\right)\right)\right] \\
-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \nabla_\mu F^{\mu\nu}F^{\mathrm{A}}\nabla_\rho F^{\rho}{}_{\nu} \\
+\frac{1}{2}\phi\left(\Box - m^2\right)\phi + \phi\left(\Box - m^2\right)F^{\phi}\left(\Box - m^2\right)\phi,
\end{aligned}
\tag{35}
$$

where the analytic functions of the d’Alembertian operator F^{g}, F^A , and F^{ϕ} and the second rank tensors $T_{\mu\nu}^A$ and $T_{\mu\nu}^{\phi}$ are defined as follows:

其中达朗贝尔算符的解析函数 F^{g}, F^A 和 F^{ϕ} ，以及二阶张量 $T_{\mu\nu}^A$ 和 $T_{\mu\nu}^{\phi}$ 定义如下:

$$
\begin{aligned}
F_{\mathrm{g}}^{\mu\upsilon,\rho\sigma} \equiv \left(g^{\mu\rho}g^{\upsilon\sigma} - \frac{1}{2}g^{\mu\upsilon}g^{\rho\sigma}\right)\left(\frac{e^{H_{\mathrm{g}}(\Box)} - 1}{\Box}\right), \\
F^{\mathrm{A}} \equiv \frac{1}{2}\left(\frac{e^{H_A(\Box)} - 1}{\Box}\right) \\
F^{\phi} \equiv \frac{1}{2}\left(\frac{e^{H_{\phi}(\Box - m^2)} - 1}{\Box - m^2}\right), \\
T_{\mu\nu}^A \equiv F_{\mu\sigma}F_{\nu}^{\sigma} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \\
T_{\mu\nu}^{\phi} \equiv \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(\partial_\lambda\phi\partial^\lambda\phi + m^2\phi^2\right).
\end{aligned}
\tag{36}
$$

H_{g}, H_A , and H_{ϕ} are form factors suitably chosen so that the theory is unitary and finite at quantum level in odd dimension (in particular in $D = 5$). In particular, the theory (35) has the same spectrum of the equivalent local theory, namely, the graviton, the photon, and the real scalar field (see [4, 6, 7] for more details), and by the field redefinition theorem all the tree-level n -point functions for the theory (35) are identical to the ones in local Einstein-Hilbert gravity coupled to the local Maxwell field and a local scalar field. It is straightforward to prove that the theory above satisfies the field redefinition theorem, namely, it is equivalent to Einstein's gravity minimally coupled to the electromagnetic field and scalar matter. Therefore, all the tree-level n -point functions for the theory (35) are identical to the ones that one can compute in local Einstein-Hilbert gravity couple to the local Maxwell field and a local scalar field. In particular we can consider the elastic scattering of gravitons on massive scalars, and the photon-graviton scattering, whose amplitudes read

H_g, H_A 和 H_ϕ 是经过适当选取的形状因子，保证该理论在奇数维 (特别是在 $D=5$ 中) 幺正且量子层级有限。特别地，理论 (35) 与等价局域理论具有完全相同的能谱，即引力子、光子和实标量场 (详见 [4, 6, 7])，并且根据场重定义定理，理论 (35) 所有树图层级 n 点函数都等价于局域爱因斯坦-希尔伯特引力耦合局域麦克斯韦场和局域标量场的对应点函数。不难证明上述理论满足场重定义定理，即它等价于最小耦合电磁场和标量物质的爱因斯坦引力。因此，理论 (35) 所有树图层级 n 点函数都完全等同于局域爱因斯坦-希尔伯特引力耦合局域麦克斯韦场和局域标量场中可计算得到的结果。特别地，我们可以考虑引力子在大质量标量上的弹性散射，以及光子-引力子散射，它们的振幅为

$$\mathcal{A}(h,\phi;h,\phi)_{2;2} = i8\pi G \frac{\left(m^4 - su\right)^2}{t\,(s+m^2)\,(u+m^2)},$$

$$\mathcal{A}(h,\phi;h,\phi)_{-2;2} = i8\pi G \frac{m^4 t}{(s+m^2)\,(u+m^2)}, \tag{37}$$

$$\mathcal{A}(h,A;h,A)_{1,2;1,2} = -i8\pi G \frac{u^2}{t},$$

$$\mathcal{A}(h,A;h,A)_{1,-2;1,-2} = -i8\pi G \frac{s^2}{t}, \tag{38}$$

plus the ones one can obtain by parity conjugation.

加上宇称共轭可得到的其余振幅。

Taking the massless limit of (37), the helicity flip amplitude vanishes, while

对 (37) 取无质量极限后，螺旋度翻转振幅消失，同时

$$\mathcal{A}(h,\phi;h,\phi)_{2;2} = i8\pi G \frac{su}{t}. \tag{39}$$

All the above amplitudes in the eikonal limit $s \ll t$ simplify to

eikonal 极限 $s \ll t$ 下，上述所有振幅可化简为

$$-i8\pi G \frac{s^2}{t} \tag{40}$$

and the time delay is the same we have computed for the four-graviton amplitudes. Therefore, the non-local theory (35) is causal as well as the local Einstein-Maxwell-scalar theory.

且时滞与我们计算四引力子振幅得到的结果一致。因此，非局域理论 (35) 和局域爱因斯坦-麦克斯韦-标量理论一样是因果的。

One could wonder whether causality can still be preserved in a theory where the nonlocality explicitly shows up in the amplitudes, i.e., in cases where the field redefinition theorem cannot be applied. An indication that this is actually possible comes from the theory whose action consists of (2) and the minimally coupled ordinary two-derivative scalar matter

人们可能会好奇，如果非局域性明确出现在振幅中，也就是无法应用场重定义定理的情况，理论还能保持因果性吗。实际存在这种情况的一个线索来自如下理论: 其作用量由 (2) 加上最小耦合的普通二阶导数标量物质构成，即

$$S = S_{\mathrm{g}} + \int d^D x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right). \tag{41}$$

Using the graviton propagator (8), the tree-level gravitational scattering amplitude for two-scalars in two-scalars can be easily obtained (we here assume $m = 0$):

利用引力传播子 (8)，我们可以很容易得到两个标量散射到两个标量的树图层级引力散射振幅 (此处我们假设 $m = 0$):

$$\mathcal{A}_s = -i8\pi G\frac{ut}{s}e^{-H(s)}, \mathcal{A}_t = -i8\pi G\frac{s(s+t)}{t}e^{-H(t)}, \mathcal{A}_u = -i8\pi G\frac{st}{u}e^{-H(u)}.$$

(42)

In the Regge limit $t \ll s$, the leading contribution comes from the amplitude in the t -channel, namely,

在 Regge 极限 $t \ll s$ 下，领头贡献来自 t 道的振幅，即

$$\mathcal{A}_t(s,t) \approx (-1)\, i8\pi G\frac{s^2}{t}e^{-H(t)}. \tag{43}$$

For $D > 4$ there are no issues related to infrared divergences, and we can now compute the phase (25) in $D = 5$,

对于 $D > 4$ ，不存在红外发散相关问题，现在我们可以计算 $D = 5$ 中的相位 (25),

$$\begin{aligned}\delta(b,s) &= \frac{1}{2s}\int\frac{d^3\vec{q}}{(2\pi)^3}e^{i\vec{q}\cdot\vec{b}}\mathcal{A}_t\left(s,-\vec{q}^2\right) = 4\pi G s\int\frac{d^3\vec{q}}{(2\pi)^3}e^{i\vec{q}\cdot\vec{b}}\frac{e^{-H\left(-\vec{q}^2\right)}}{\vec{q}^2}\\ &= \frac{2Gs}{\pi}\int dq\frac{\sin(bq)}{bq}e^{-H(-q^2)},\end{aligned} \tag{44}$$

where $q = |\vec{q}|$. In particular, for the form factor

其中为 $q = |\vec{q}|$ 。特别地，对于形状因子

$$e^{-\sigma\Box}, \tag{45}$$

which emerges naturally in string field theory [32-35], one finds the analytic result

它在弦场论中自然出现 [32-35]，我们可以得到解析结果

$$\delta(b,s)_{\mathrm{SFT}} = Gs\frac{\mathrm{Erf}\,(b/2\ell_\Lambda)}{b}, \tag{46}$$

which reduces to the one in Einstein’s theory for $b \gg \ell_\Lambda$, namely,

当 $b \gg \ell_\Lambda$ 时，该结果退化为爱因斯坦理论中的结果，即

$$\delta(b,s)_{\mathrm{SFT}} \to \delta_{\mathrm{EH}}\,(b,s) = \frac{Gs}{b}. \tag{47}$$

The corresponding time delays are

对应的时滞为

$$\Delta t_{\mathrm{SFT}} = \frac{16EG}{\pi}\pi\frac{\mathrm{Erf}\,(b/2\ell_\Lambda)}{b}, \tag{48}$$

$$\Delta t_{\mathrm{EH}} = \frac{16EG}{\pi}\frac{\pi}{b}. \tag{49}$$

In Fig. 1 we plot Δt_{T} for the form factor (5), which has been obtained numerically, together with Δt_{SFT} and Δt_{EH} . Very similar results can be obtained for different values of α and γ .

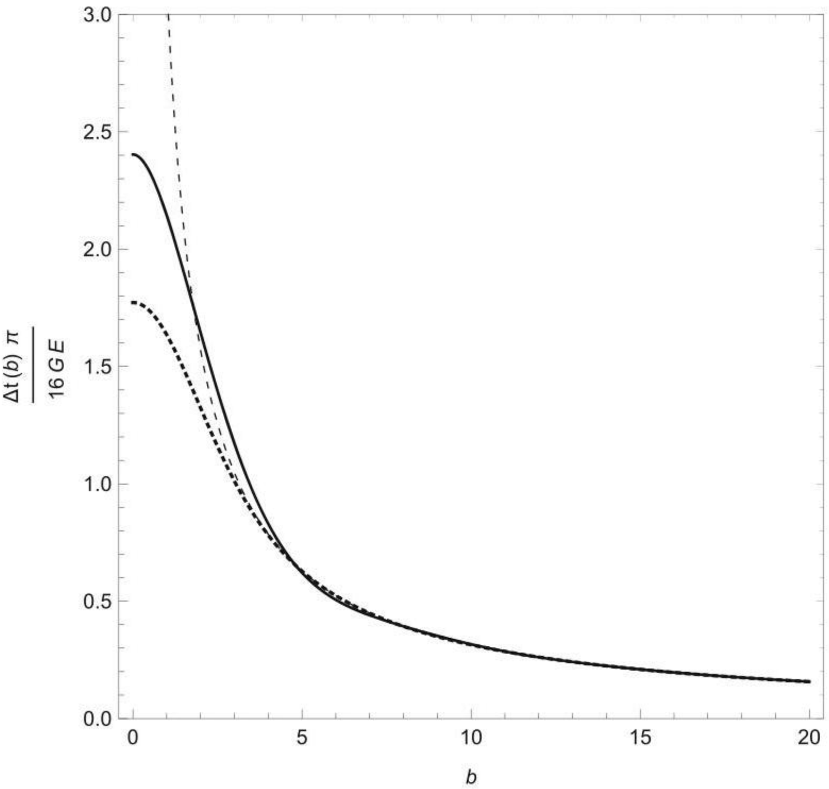
在图 1 中，我们绘制了通过数值计算得到的形状因子 (5) 对应的 Δt_{T} ，同时也绘制了 Δt_{SFT} 和 Δt_{EH} 。针对不同的 α 和 γ 取值，也可以得到非常相似的结果。

From the analytical results as well as from the plots, it is clear that the Shapiro’s time delay never becomes negative, and the causality condition is satisfied up to and beyond the nonlocality scale ℓ_Λ .

从解析结果和图线中都可以明确看出，夏皮罗时间延迟始终为正，因果性条件在非定域标度 ℓ_Λ 之内及超出该标度的范围都满足。

Fig. 1 From top to bottom the lines represent respectively the following Shapiro’s delays: $\Delta t_{\mathrm{EH}}, \Delta t_{\mathrm{T}}$ (for $\gamma = 3$), and Δt_{SFT} . We also assumed $\ell_\Lambda = 1$

图 1 从上到下，各线依次代表如下夏皮罗时间延迟: $\Delta t_{\mathrm{EH}}, \Delta t_{\mathrm{T}}$ (对应 $\gamma = 3$)，以及 Δt_{SFT} 。我们还假设了 $\ell_\Lambda = 1$



Supersymmetry is another powerful tool to couple nonlocal gravity to matter in a way consistent with unitarity, finiteness, but also causality. Indeed, in [31] we constructed the $N = 1$ nonlocal supergravity that has the same tree-level scattering amplitudes as the local one. Therefore, on the basis of the theorem reviewed in this section, the theory is causal.

超对称是另一种将非定域引力与物质耦合的有力工具，能让耦合满足幺正性、有限性，同时也满足因果性。事实上，在文献 [31] 中我们构建了 $N = 1$ 非定域超引力，它与定域超引力拥有完全相同的树图散射振幅。因此，基于本节梳理的定理，该理论满足因果性。

Local Lee-Wick quantum gravity Recently a local higher derivative theory has been proposed as a good candidate for a UV completion of the Einstein-Hilbert theory [36, 37] . The theory has no real ghosts in the spectrum, but allows for complex conjugate ghosts. It turns out that it is unitary at tree-level [36, 37] and also at any perturbative order in the loop expansion [38-40]. Moreover, it is super-renormalizable or finite at quantum level [36, 37] . Notice that the S -matrix is unitary in the subspace of real states as a consequence of the energy conservation and on the basis of the empirical evidence that complex energy is not realized in nature.

定域 Lee-Wick 量子引力近来，一种定域高阶导数理论被提出，作为爱因斯坦-希尔伯特理论 [36, 37] 紫外完备的优秀候选。该理论谱中不存在实鬼粒子，但允许复共轭鬼粒子。研究表明，它在树图级 [36, 37] 以及圈展开的任意微扰阶 [38-40] 都满足幺正性。此外，它在量子水平是超可重整化或有限的 [36, 37] 。需要注意的是，由于能量守恒，且根据复能量不会在自然界中实现的经验事实，S 矩阵在实态子空间中是幺正的。

The minimal theory in which only the graviton propagates and a pair of complex conjugate ghosts reads

仅存在引力子传播和一对复共轭鬼粒子的最小理论形式为

$$S_{\mathrm{g}}=\frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\left[R+\sigma^2 G_{\mu\nu}\Box R^{\mu\nu}+V(\mathcal{R})\right]. \tag{50}$$

The propagator of the theory (50) shows up two complex conjugate poles:

理论 (50) 的传播子存在两个复共轭极点:

$$G(k)=\frac{1}{i(k^2-i\varepsilon)(1+\sigma^2 k^4)}\left(P^{(2)}-\frac{1}{D-2}P^{(0)}\right). \tag{51}$$

If the potential $V(\mathcal{R})$ is at least quadratic in the Ricci tensor, the tree-level amplitudes coincide with those in Einstein gravity (this is a consequence of the theorem in the previous section), and causality is not violated. When the Lee-Wick gravity (50) is coupled to the scalar matter (41), the tree-level gravitational scattering amplitude for two scalars in two scalars is obtained from the amplitude in the t -channel:

如果势 $V(\mathcal{R})$ 在里奇张量中至少是二次的，那么树图振幅就与爱因斯坦引力的振幅一致 (这是上一节定理的推论)，不会违反因果性。当 Lee-Wick 引力 (50) 与标量物质 (41) 耦合时，我们可以从 t 道的振幅得到两个标量的弹性树图引力散射振幅:

$$\mathcal{A}_t=-i8\pi G\frac{s(s+t)}{t(1+\sigma^4 t^2)}\approx -i8\pi G\frac{s^2}{t(1+\sigma^4 t^2)}. \tag{52}$$

Notice that $t\ll s$, but t can be larger than Λ^2 . Replacing the amplitude (52) in (44), we get

需要注意 $t\ll s$ ，但 t 可以大于 Λ^2 。将振幅 (52) 代入 (44)，我们得到

$$\delta(b,s)_{\mathrm{SFT}}=Gs\frac{1-e^{-\frac{b}{2\ell_\Lambda}}\cos\left(\frac{b}{\sqrt{2}\ell_\Lambda}\right)}{b}. \tag{53}$$

The phase (53) is always positive, and the plot is very similar to the one of nonlocal gravity. Therefore, the Shapiro's time delay is also positive, and there is no causality violation.

相位 (53) 始终为正，图线与非定域引力的结果非常相似。因此，夏皮罗时间延迟也为正，不存在因果性违反。

Nonlocal gravity in Weyl basis Finally, we would like to present a theory that can potentially violate causality. The Lagrangian reads [5, 8-11]

外尔基下的非定域引力最后，我们介绍一个可能违反因果性的理论。其拉格朗日量为 [5, 8-11]

$$\begin{aligned}\mathcal{L}_W&=\frac{1}{2\kappa_D^2}\left(R+C_{\mu\upsilon\rho\sigma}\gamma_{\mathrm{C}}(\Box)C^{\mu\upsilon\rho\sigma}+R\gamma_{\mathrm{R}}(\Box)R\right),\\ \gamma_{\mathrm{C}}&=\frac{D-2}{4(D-3)}\frac{e^{H_2}-1}{\Box},\gamma_{\mathrm{S}}=-\frac{D-2}{4(D-1)}\frac{e^{H_0}-1}{\Box}.\end{aligned} \tag{54}$$

The theory (54) violates causality for $H_2 = H_0 = H_K$ or $H_2 = H_0 = \sigma\Box$. Indeed, expanding γ_C in Taylor series, we get also a Riemann square operator, $R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}$, that gives the same causality violation as computed in (33). On the other hand, the same theory with entire functions $H_2 = H_0 = H_T$ does not give a Shapiro’s time advanced as shown in [18].

理论 (54) 在 $H_2 = H_0 = H_K$ 或 $H_2 = H_0 = \sigma\Box$ 的情况下违反因果性。事实上，将 γ_C 展开为泰勒级数后，我们会得到一个黎曼平方算子 $R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}$ ，它给出的因果性违反与 (33) 中计算的结果相同。另一方面，如文献 [18] 所示，采用整函数 $H_2 = H_0 = H_T$ 的同一理论不会给出夏皮罗时间超前，即不违反因果性。

Conclusions

结论

Weakly nonlocal theories are an interesting arena where such crucial ideas about quantum gravity as ultraviolet finiteness, perturbative unitarity, and causality can be tested in a very straightforward way thanks to the powerful formalism of quantum field theory. In particular, we have given evidence that in a lot of case we can extend Einstein-Hilbert gravity without violating the notion of causality related to Shapiro’s time delay and discussed in [18]. Contrary to what happens in weakly coupled string theory, causality is not achieved by the introduction of an infinite tower of massive higher spin fields, but by avoiding the higher-derivative terms which could cause a Shapiro time advance. This has been proven to be possible in several cases; in particular, a field redefinition theorem allows to construct a very general nonlocal theory for matter coupled to gravity compatible with causality. As a particular applications of the theorem, we have discussed the Einstein-Maxwell-scalar nonlocal field theory, which can be proven to be causal, unitary, and finite in the ultraviolet. Other examples discussed in [27] are the $N = 1$ nonlocal supergravity [31] and Lee-Wick gravity [36-40].

弱非局域理论是一个有趣的研究平台，借助量子场论的强大形式体系，我们可以在其中非常直接地检验量子引力的若干关键概念，包括紫外有限性、微扰幺正性与因果性。具体而言，我们已有证据表明，在大量情形中，我们可以扩展爱因斯坦-希尔伯特引力，且不违背文献 [18] 中讨论的、与夏皮罗时间延迟相关的因果性概念。与弱耦合弦理论不同，因果性并非通过引入无穷多的大质量高自旋场层叠实现，而是通过避免可能引发夏皮罗时间超前的高阶导数项实现。我们已经证明这在多个情形中是可行的；特别是，场重定义定理允许我们构建出满足因果性、与引力耦合的物质的十分普适的非局域理论。作为该定理的一项具体应用，我们讨论了爱因斯坦-麦克斯韦-标量非局域场论，可证明该理论是因果的、幺正的，且在紫外是有限的。文献 [27] 讨论的其他例子还有 $N = 1$ 非局域超引力 [31] 以及李-威克引力 [36-40]。

Cross-References

交叉引用

Classical and Quantum Nonlocal Gravity

经典与量子非局域引力

Gauge Invariant Renormalizability of Quantum Gravity

量子引力的规范不变可重整性

Supergravity Amplitudes, the Double Copy, and Ultraviolet Behavior

超引力振幅、双拷贝与紫外行为

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